Warm-Starting **All-Pairs Shortest Paths** with Predictions

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Jonas Schmidt

Bocconi University

ESA 2026

All-Pairs Shortest Paths (APSP)

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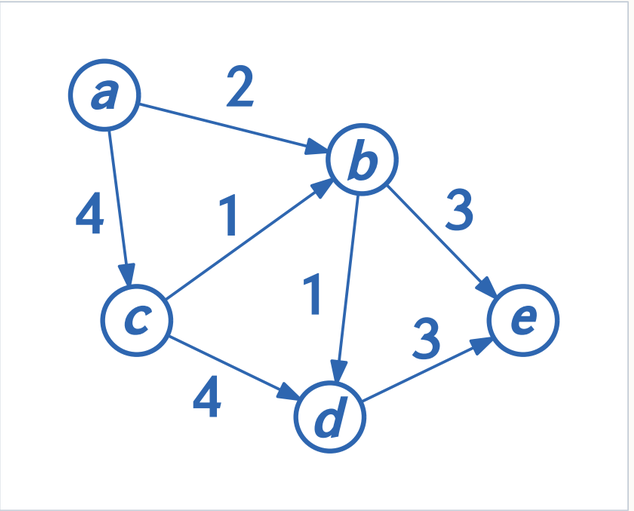

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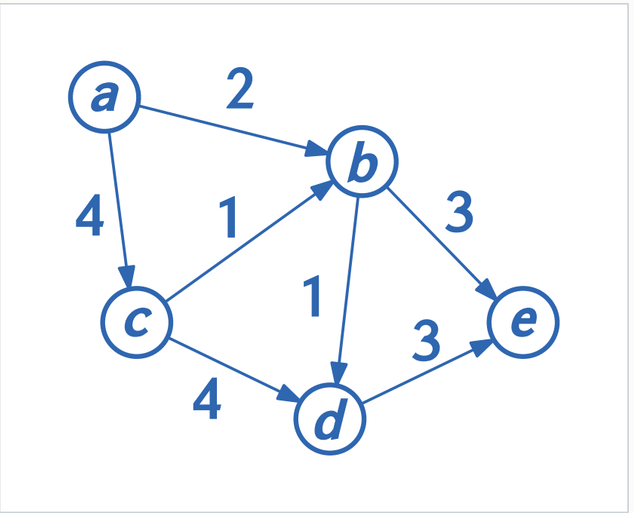

D	a	b	c	d	e
a	0	1	4	3	5
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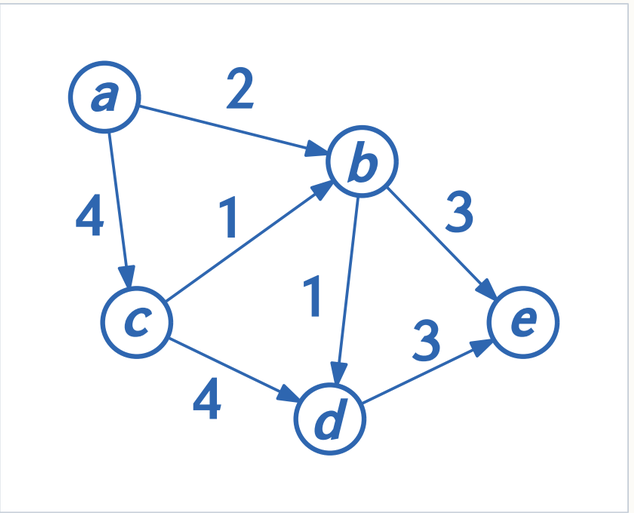
- Floyd–Warshall algorithm: $O(n^3)$ [Floyd '62] [Warshall '62]
- $n \times$ SSSP: $\tilde{O}(nm)$



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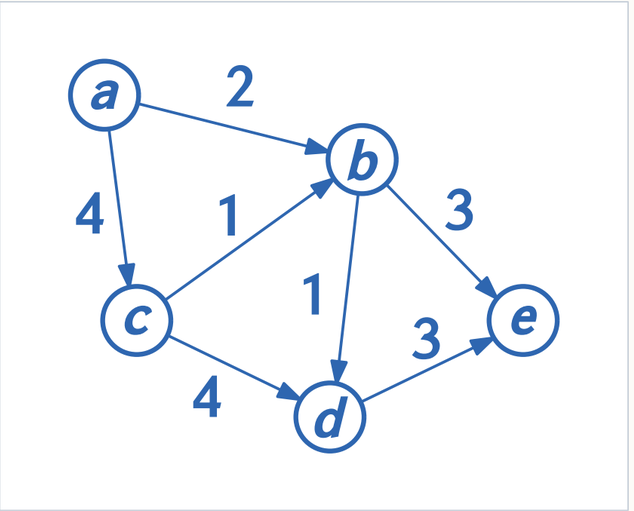

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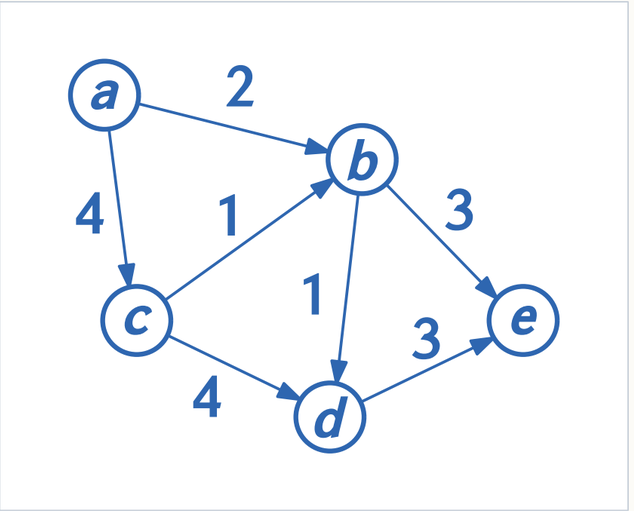
- **APSP Hypothesis:** no algorithm in time $O(n^{2.99})$



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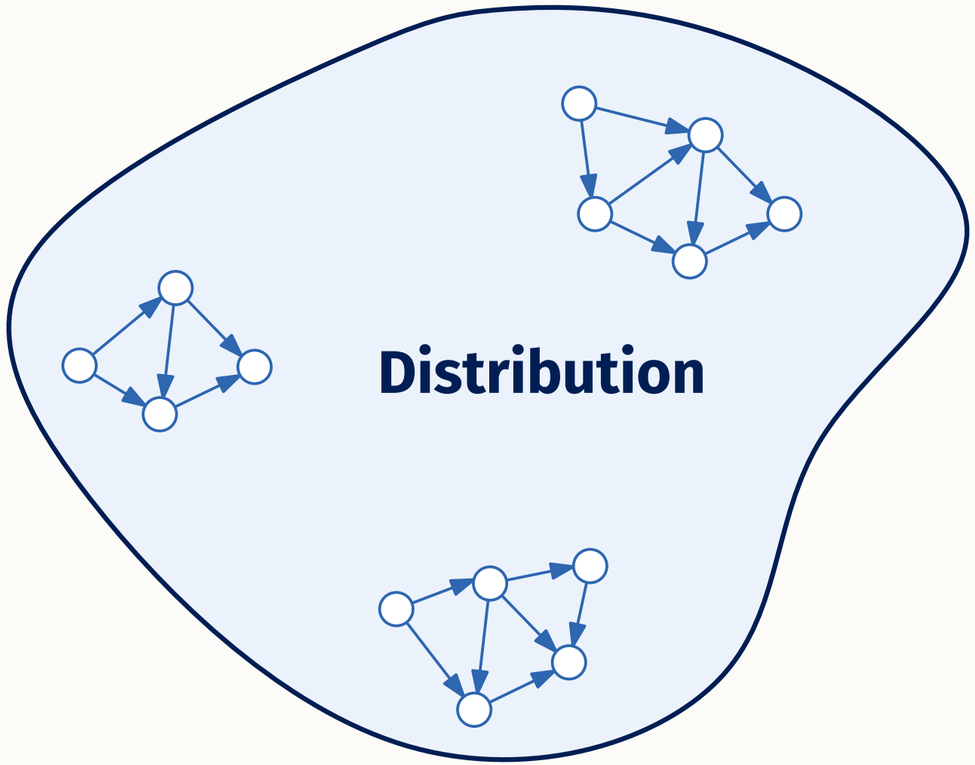

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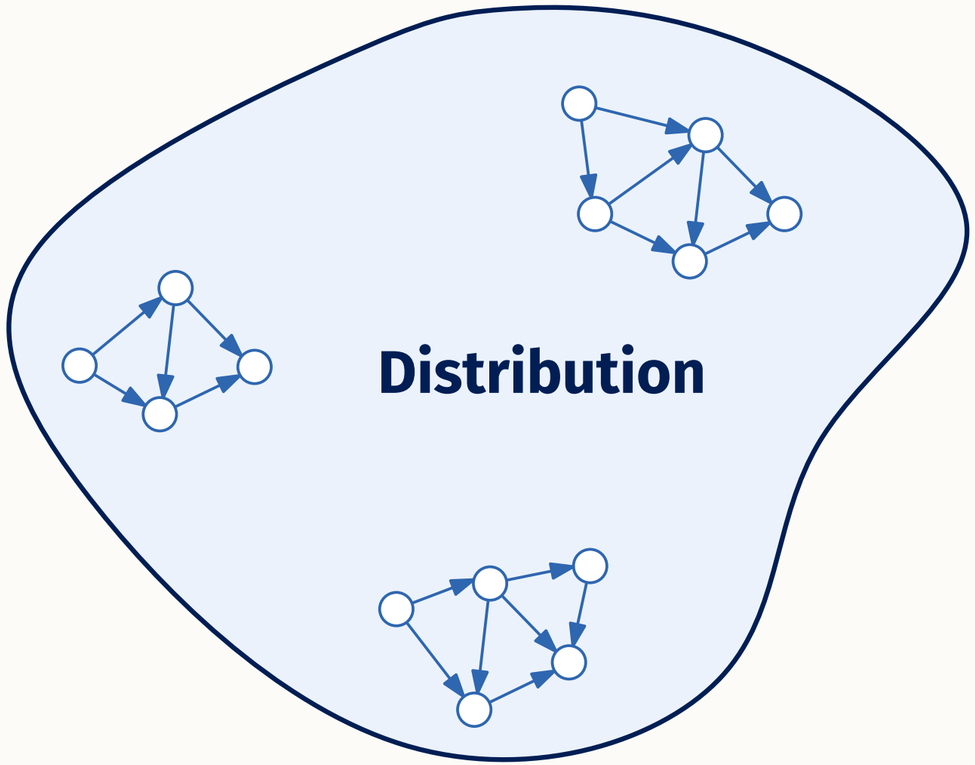

Q: Can we use **common structure** to solve instances faster (than n^3)?

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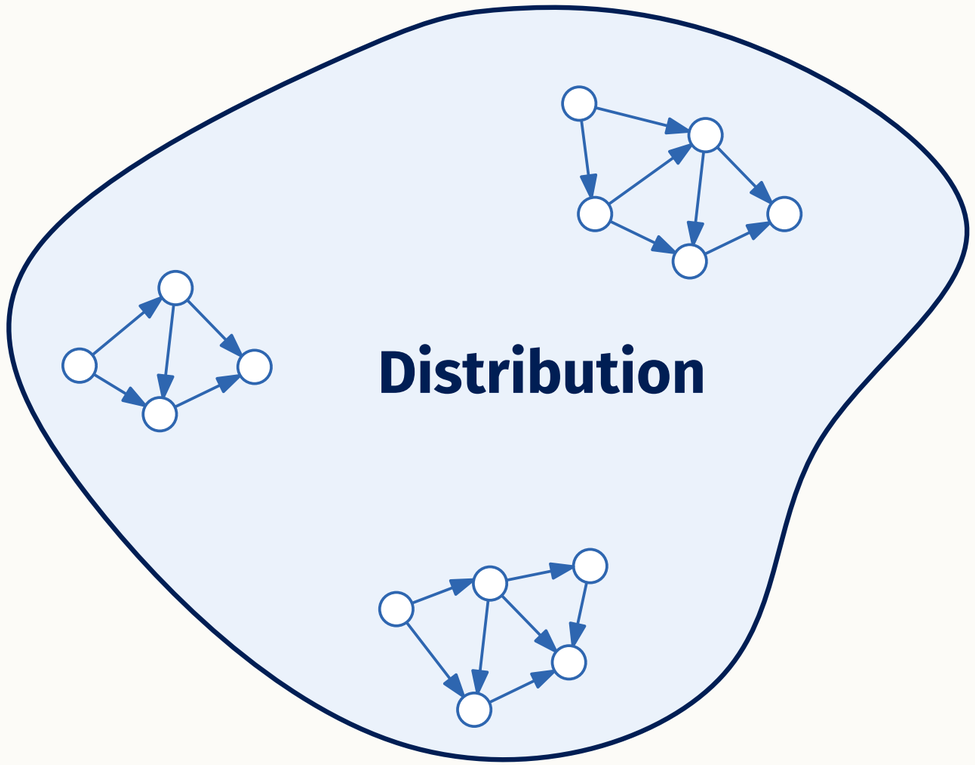

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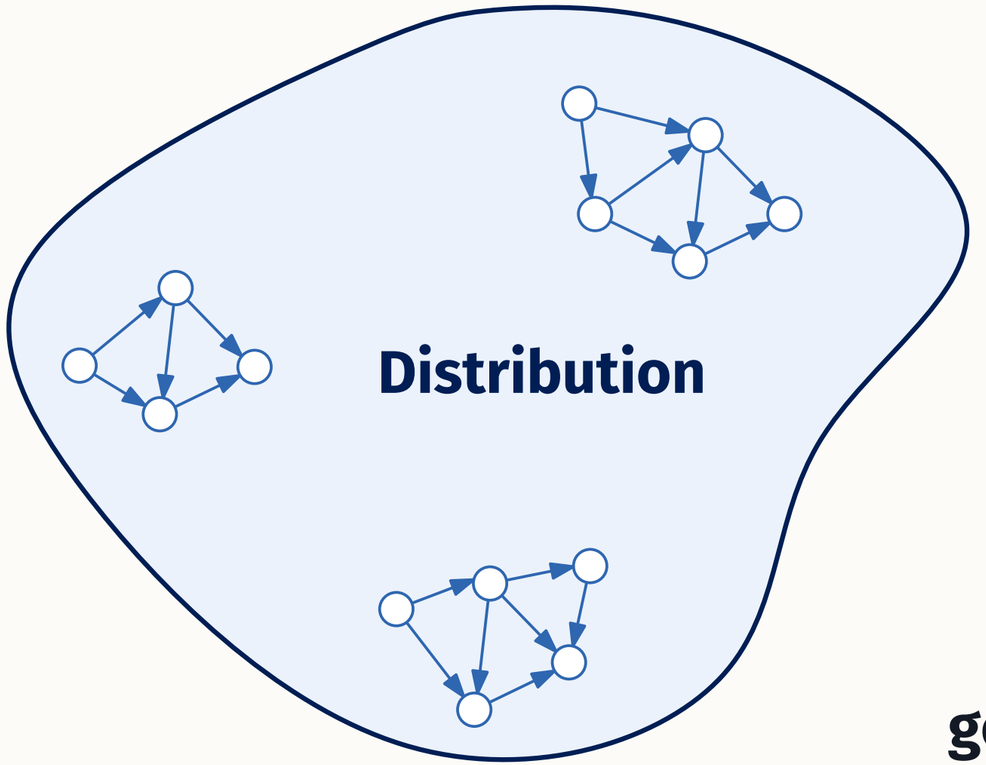

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good prediction: provable fast alg

smooth tradeoff

bad prediction: worst-case alg

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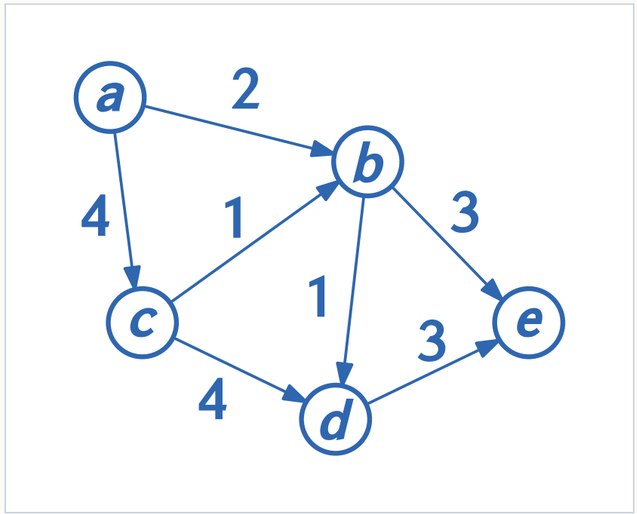

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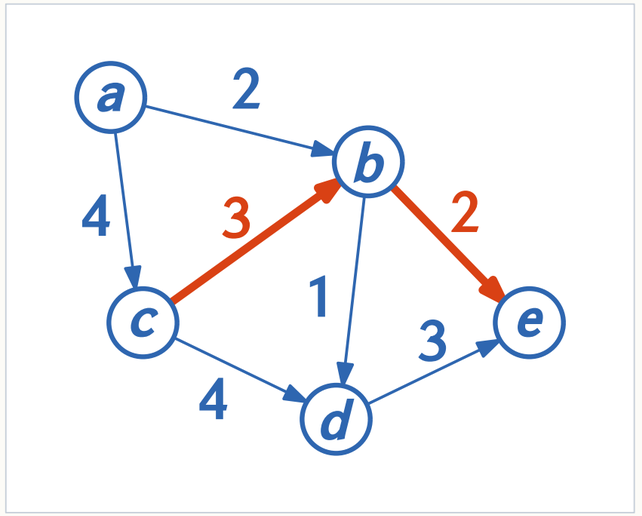

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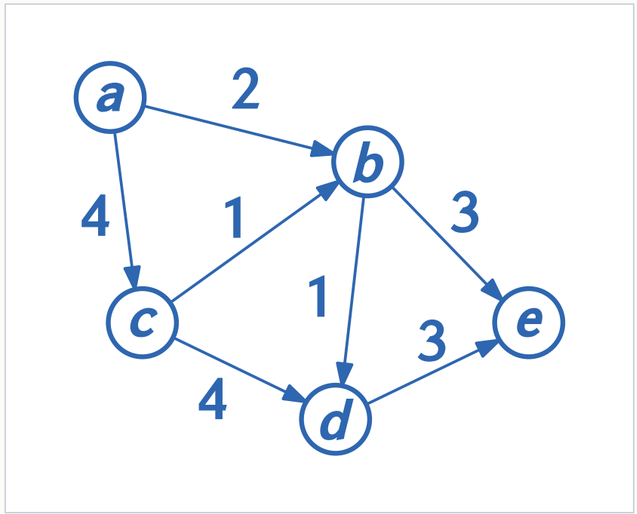

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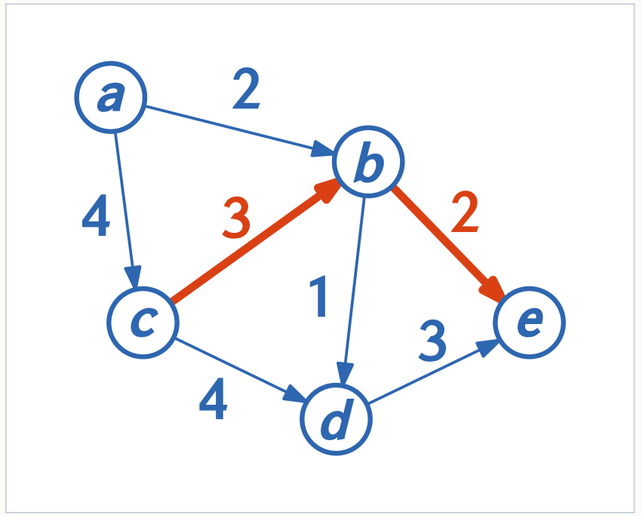

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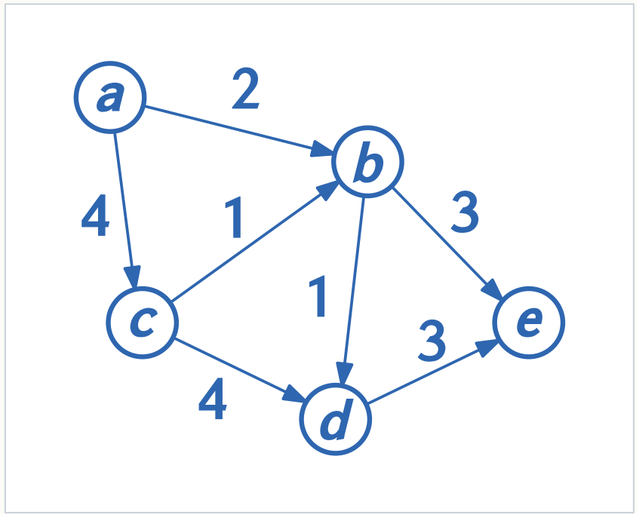
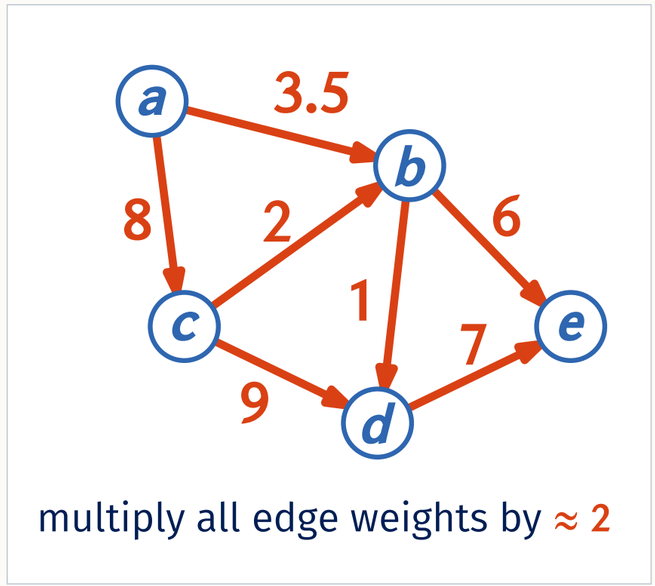


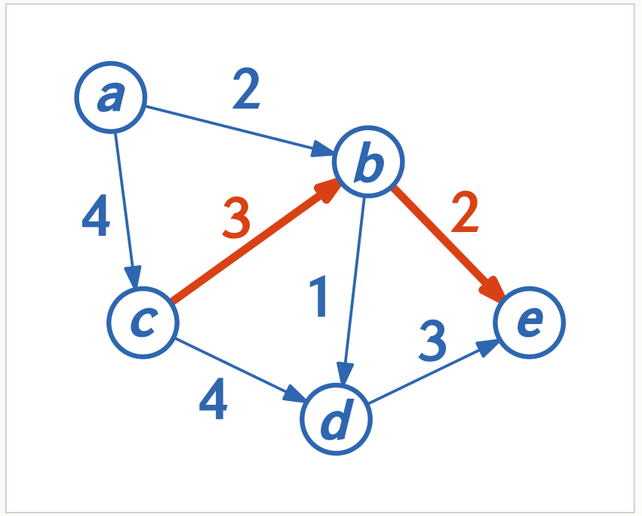
structure = **few input changes??**

different numbers, similar **path structure**

e.g., road network during rush hours



multiply all edge weights by ≈ 2



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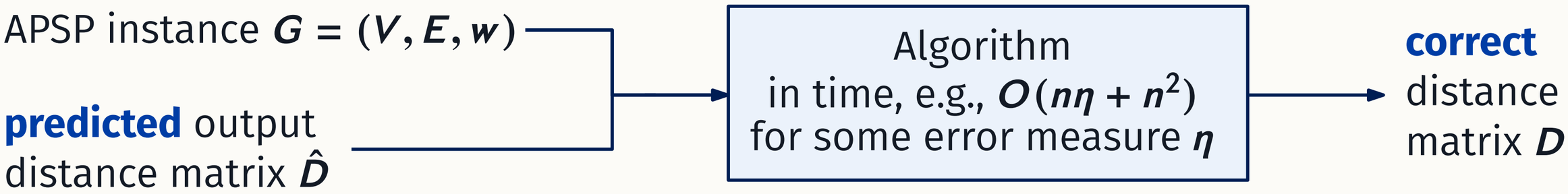

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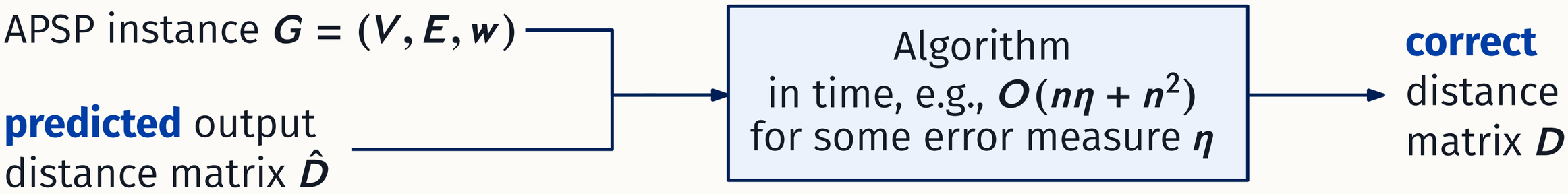

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Theorem [Vassilevska Williams, Williams '10]

If there is a $O(n^{2.99})$-time algorithm that **verifies** if a given matrix $\hat{D}$ is the correct distance matrix, the APSP hypothesis is false.

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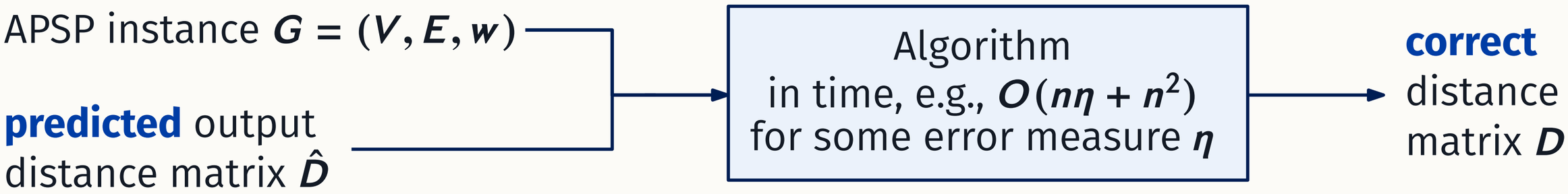

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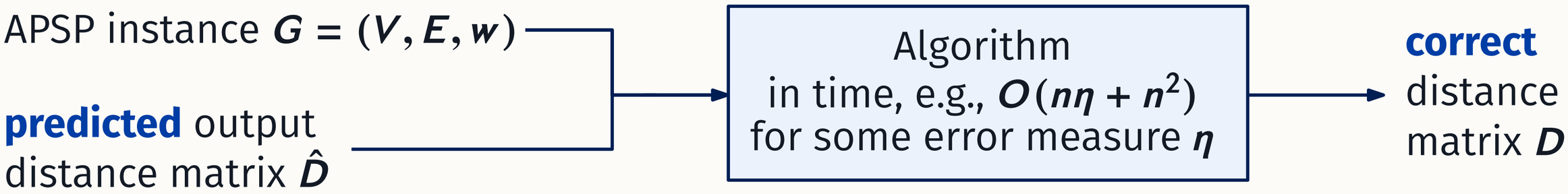

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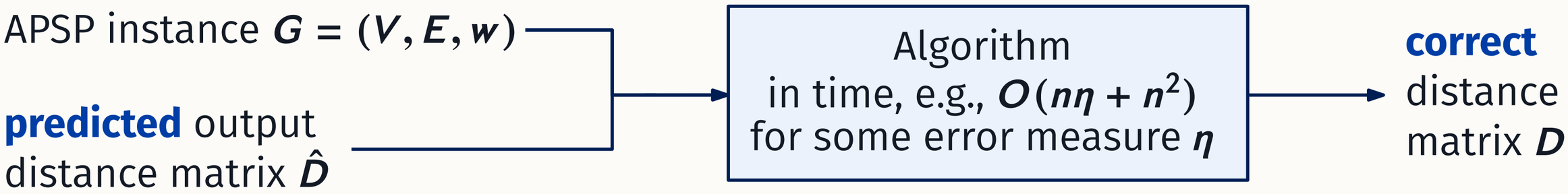

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We need a **stronger prediction**!

APSP → **APSP Verification**

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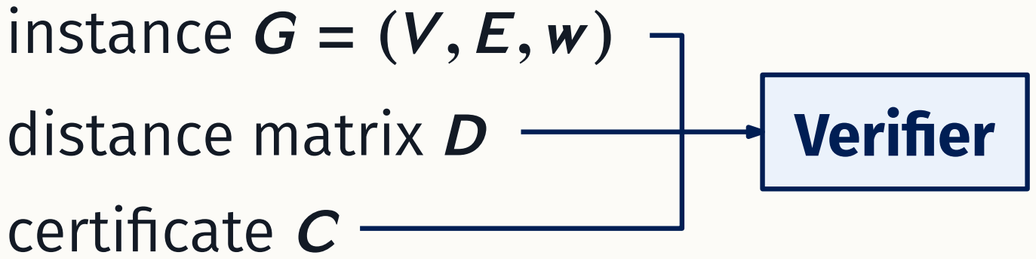

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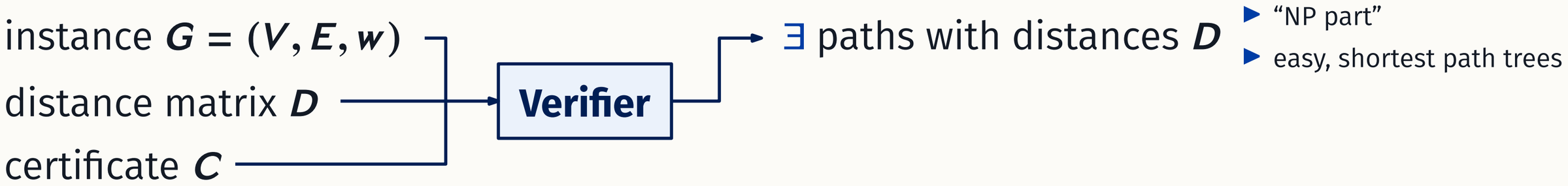

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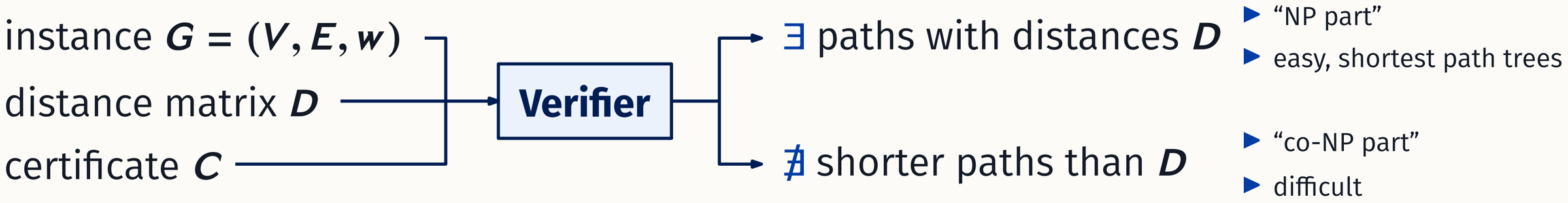

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- easy, shortest path trees

- "co-NP part"
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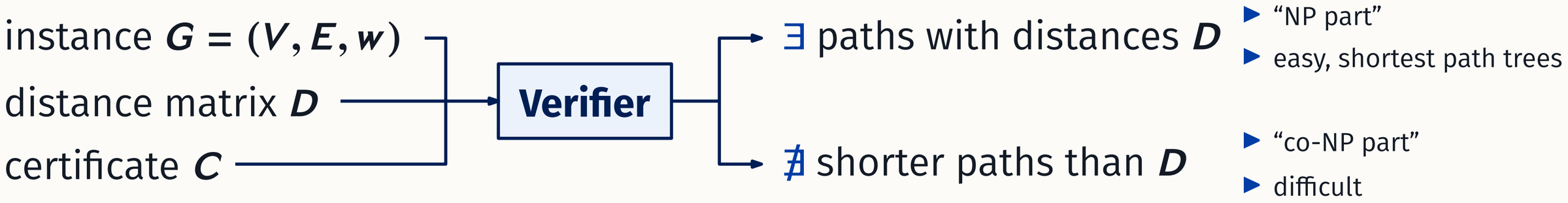

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2 co-nondeterministic algorithms known:

- Hashing-based: time $O(n^{(\omega+3)/2})$ [Carmosino, Gao, Impagliazzo, Mihajlin, Paturi, Schneider '17]
- More structural / real inputs: time $O(n^{(\omega+6)/3})$ [Chan, Vassilevska Williams, Xu '23]

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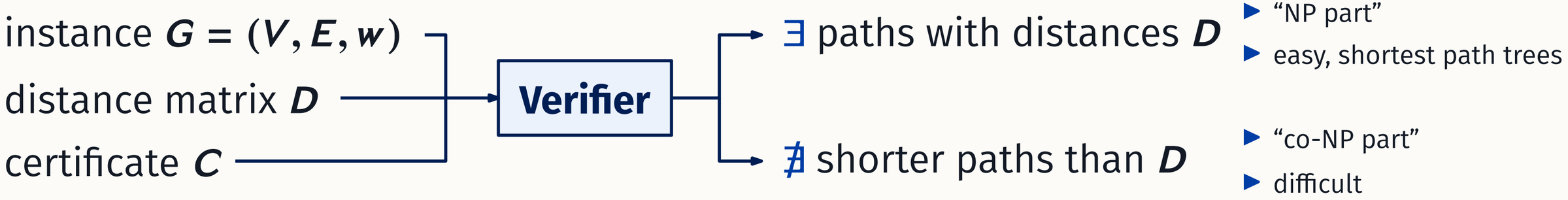

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$2 \leq \omega < 2.372$ is matrix multiplication exponent.
Assume $\omega = 2$. MM in time $O(n^{\omega})$

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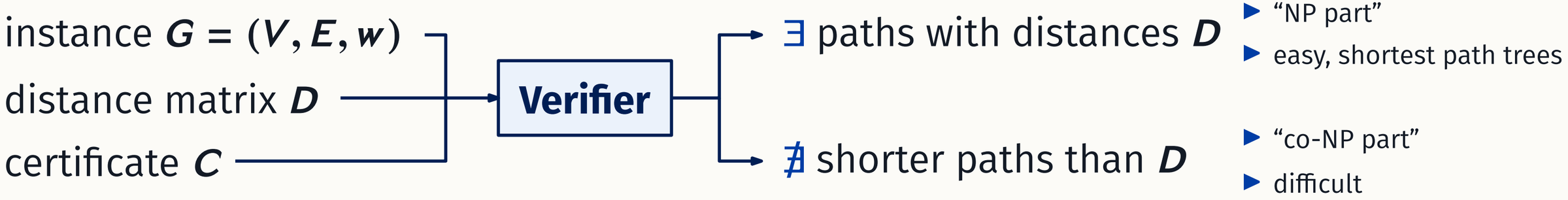

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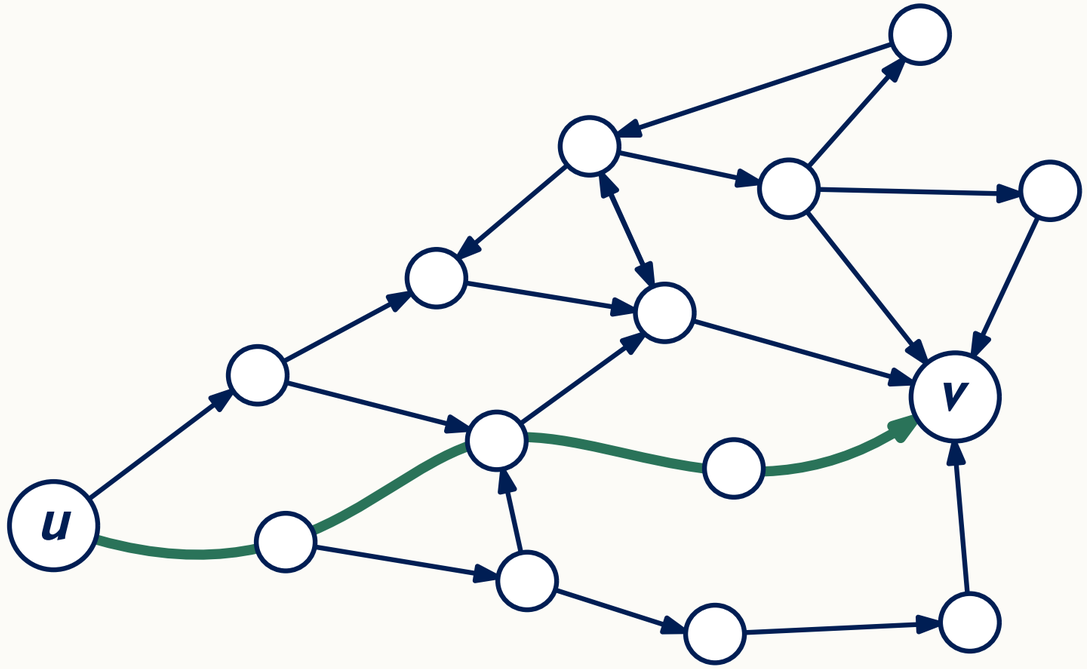

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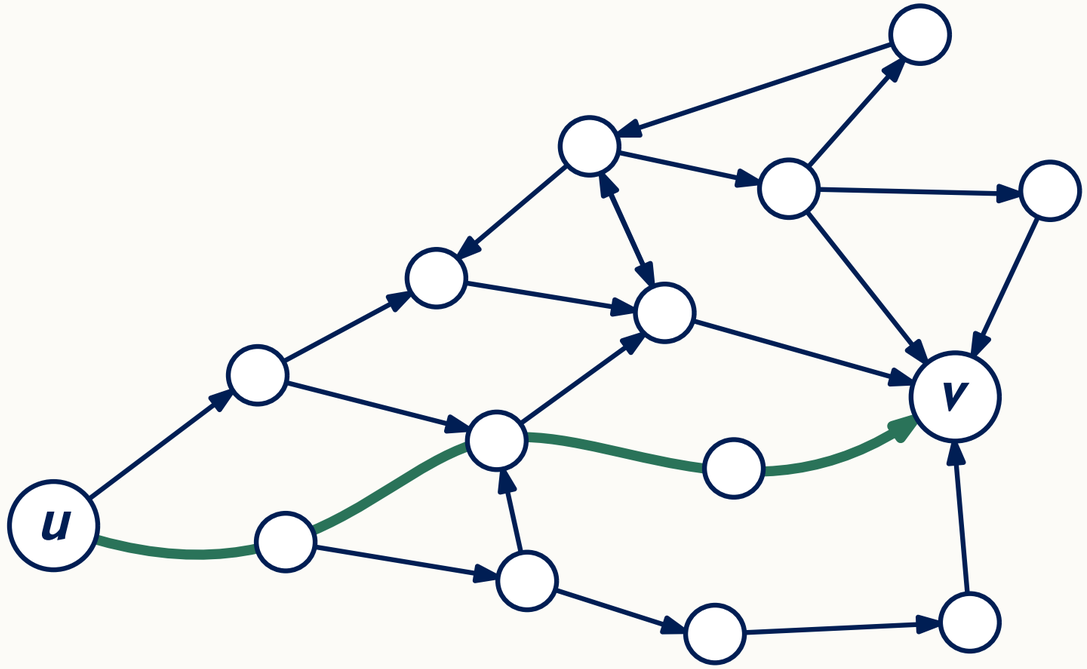
Certificate / Prediction C: (informal)

For every vertex pair (u, v), vertices **causing the shortest detour** on a u-v-path.



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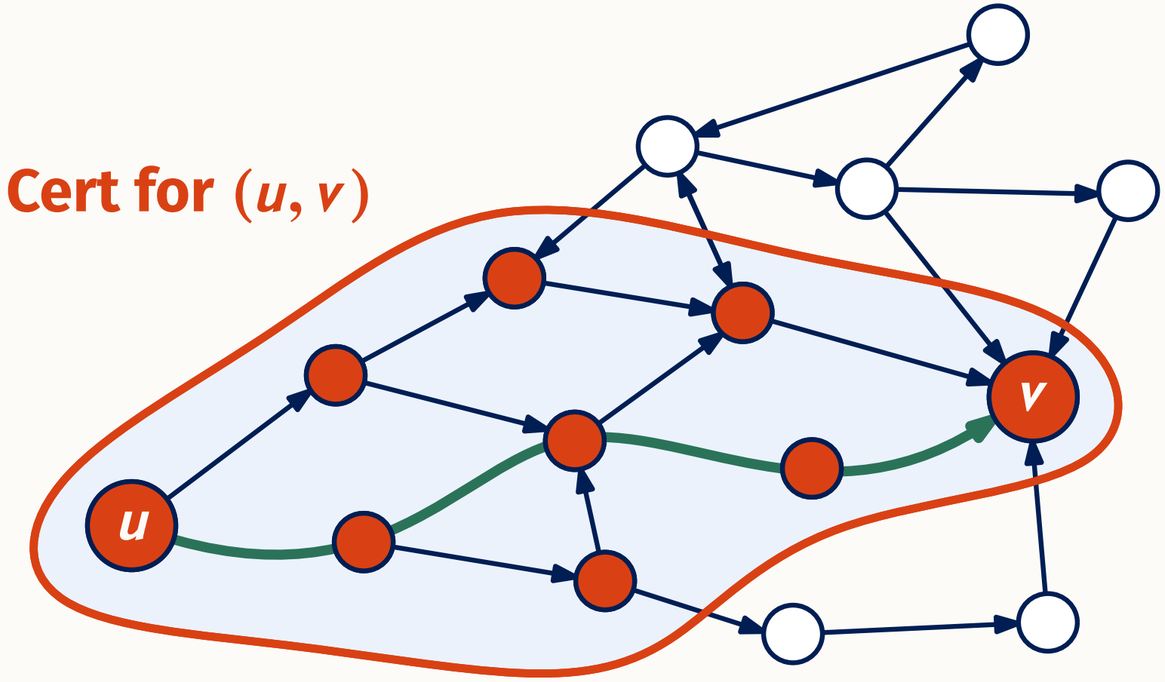

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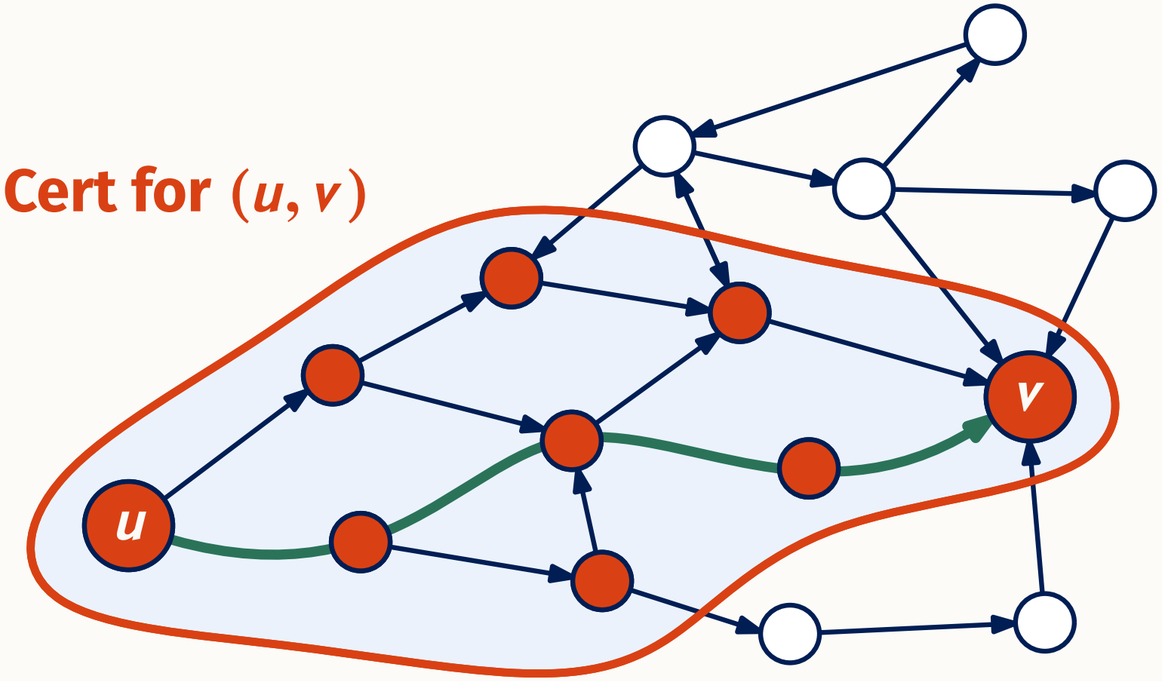

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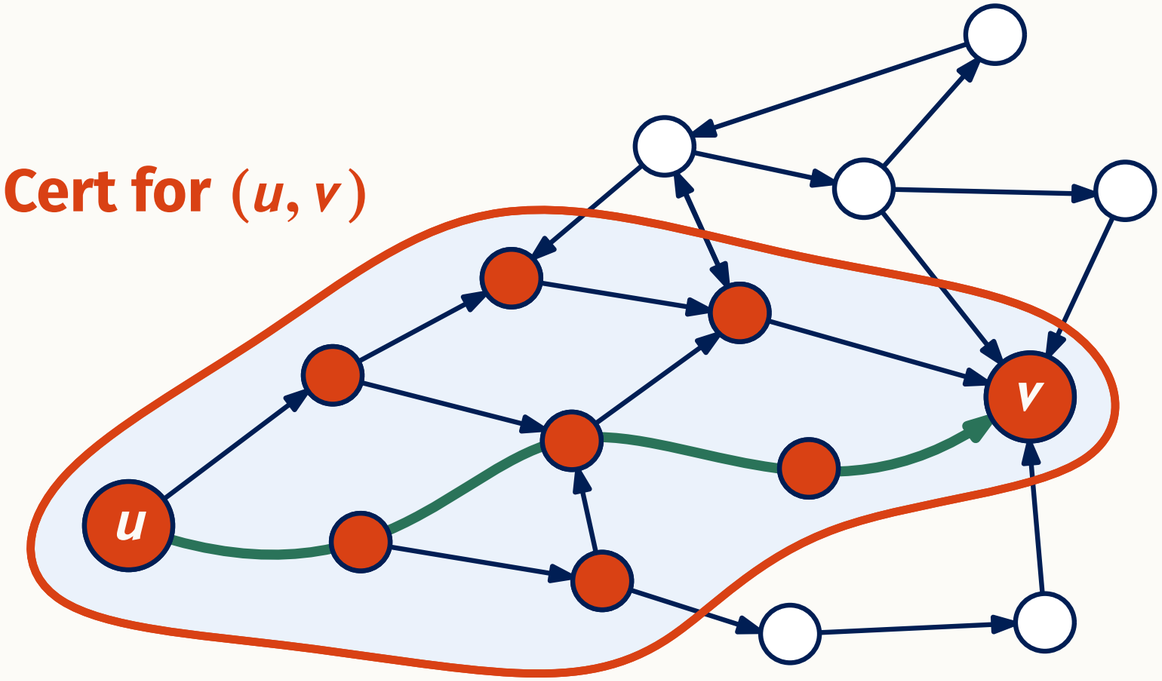

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Theorem

Given G and C, we can compute D in time $\tilde{O}(n\eta + n^{2.75})$.

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- Use the certificate C as our prediction

Certificate:

A matrix C where $C[i, j] \subseteq V$ contains the p best **detour vertices** for (i, j).

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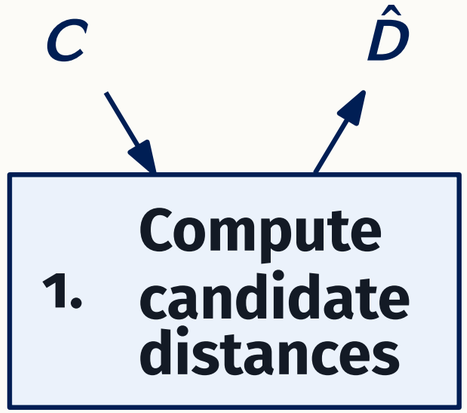

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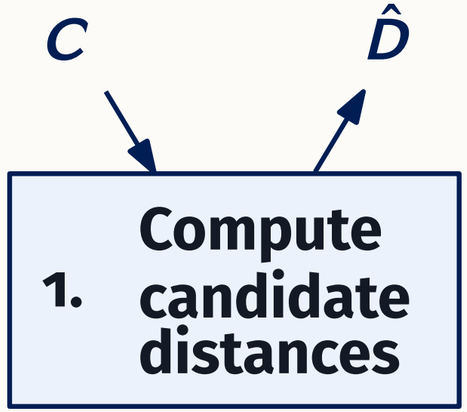

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In time $\tilde{O}(|C|)$, we can compute "good candidate distances" $\hat{D}$ from C.

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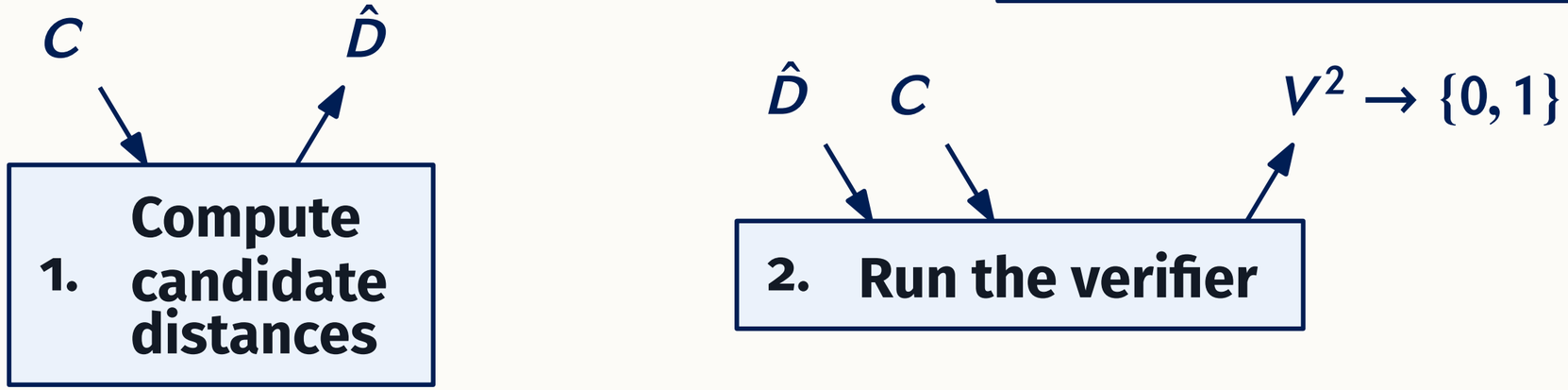

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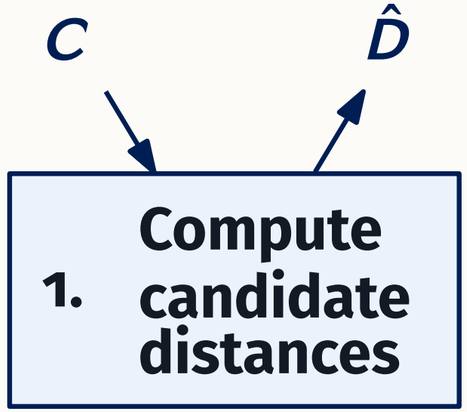

verifies each entry $\hat{D}[i,j]$ **locally**:

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Lemma (informal)

In time $\tilde{O}(|C|)$, we can compute "good candidate distances" $\hat{D}$ from C.

"perform **all and only** relaxations in C" ← in a **good order**!

Turning the Verifier Into an Algorithm with Predictions

- Use the certificate C as our prediction

3 steps:

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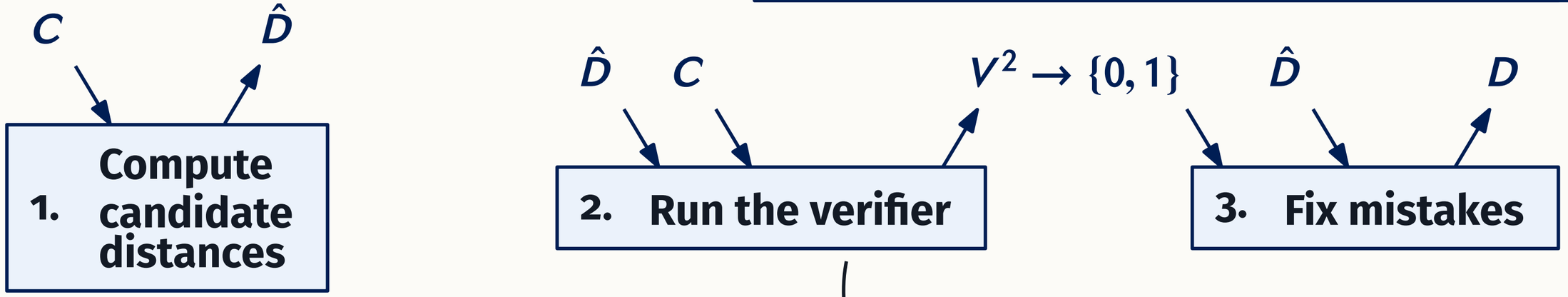

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Given G and C, we can compute D in time $\tilde{O}(n\eta + n^{2.75})$.

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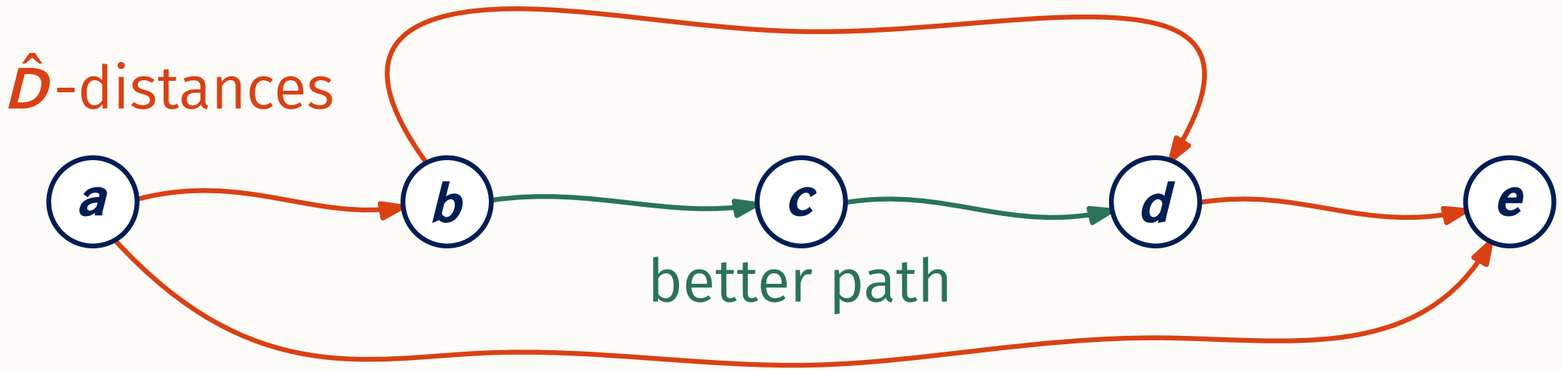

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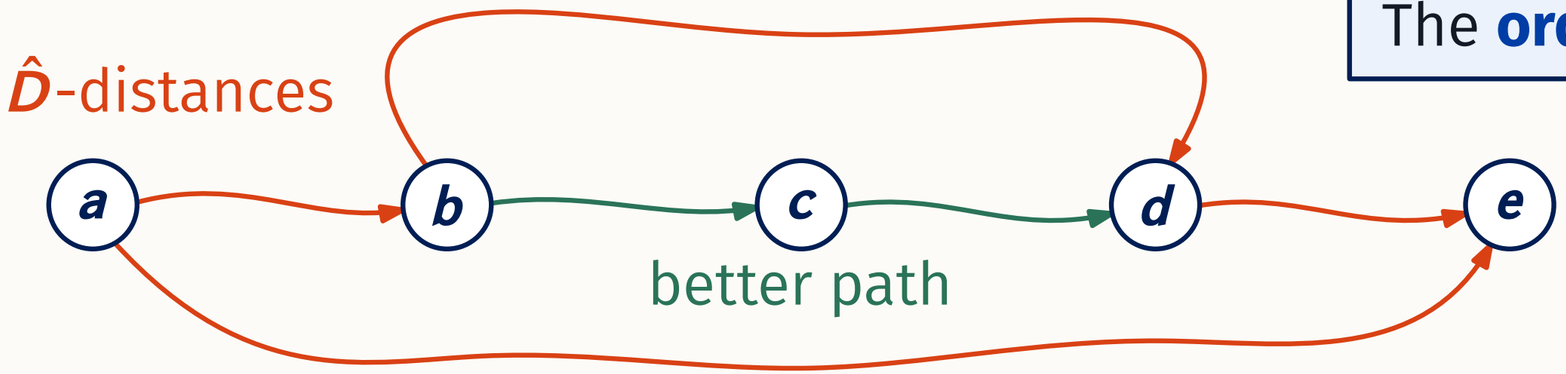

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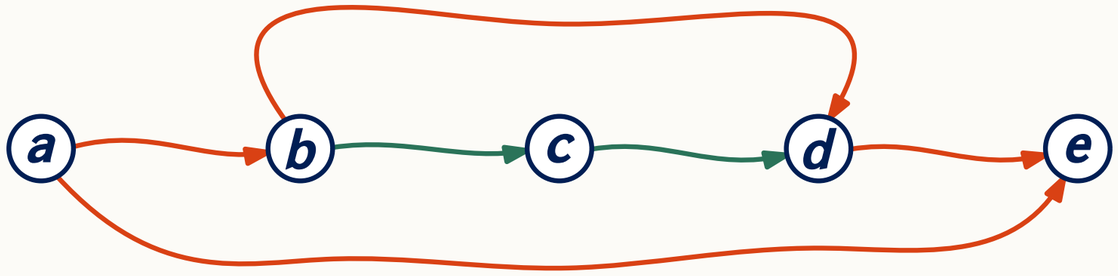

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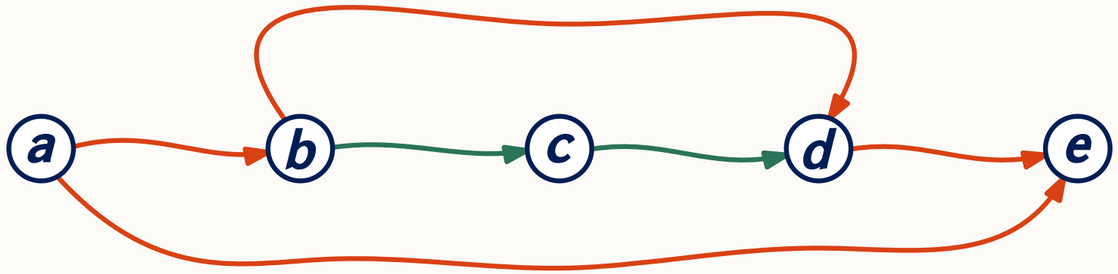

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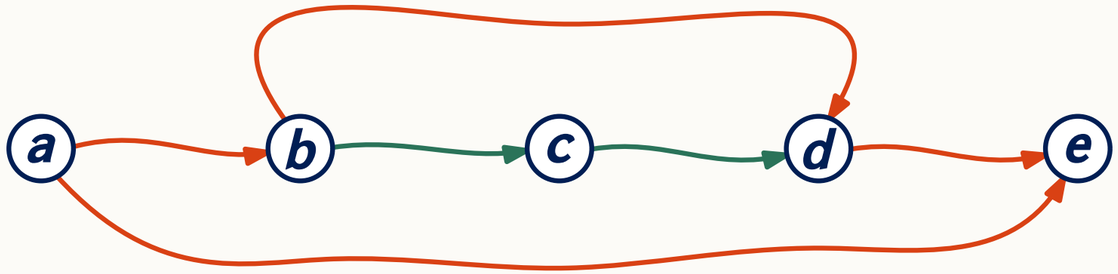

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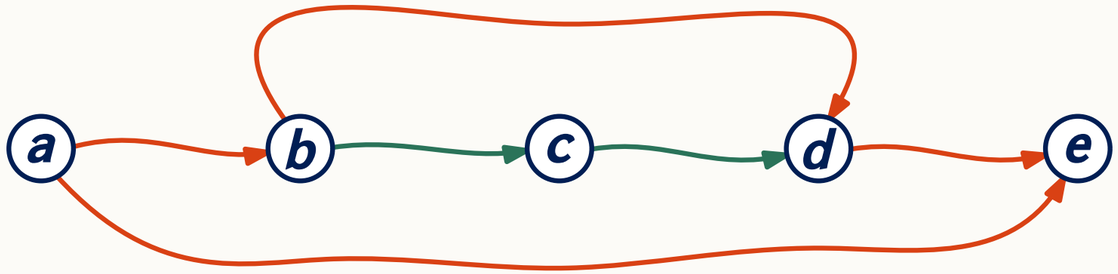

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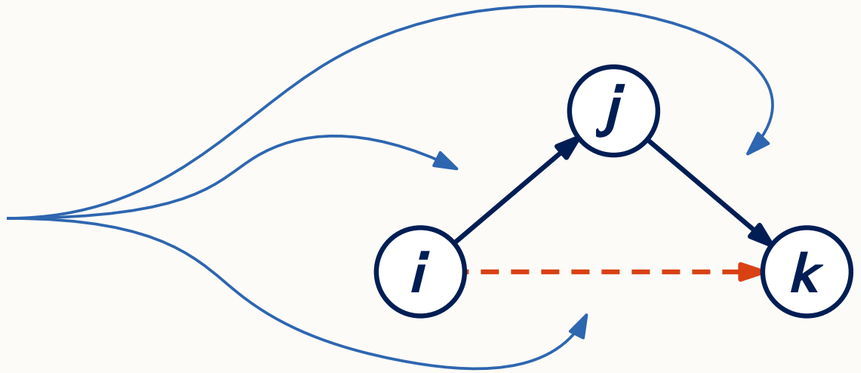

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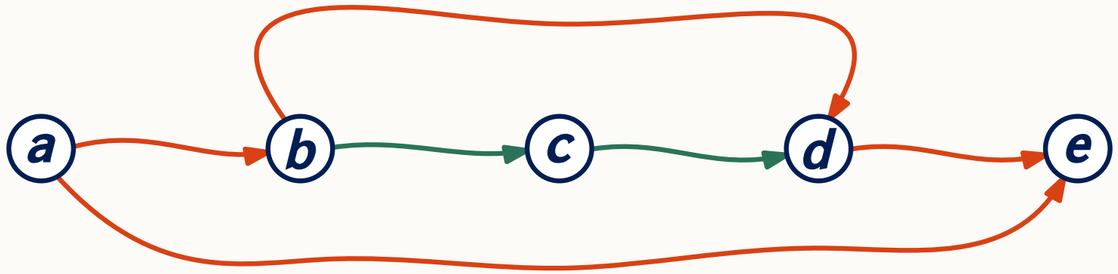

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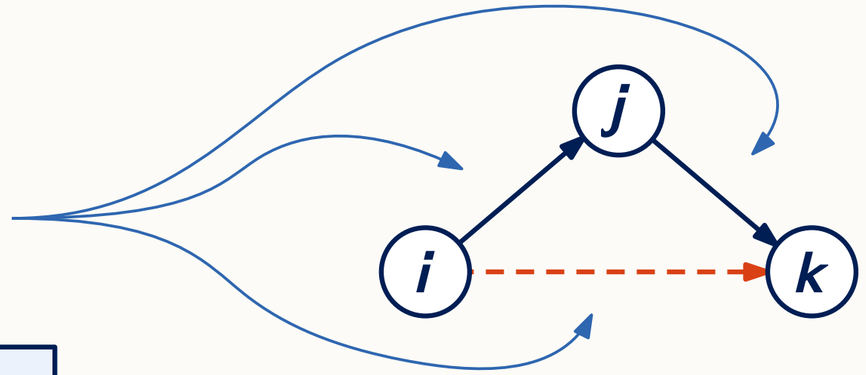

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#{ total counted pairs } $\cdot\, O(n) = O(\eta \cdot n)$

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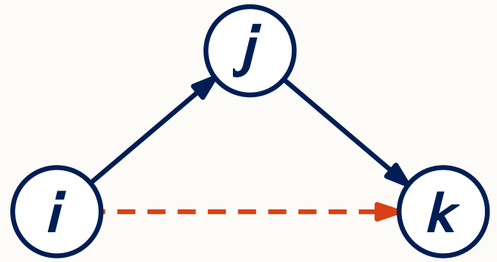

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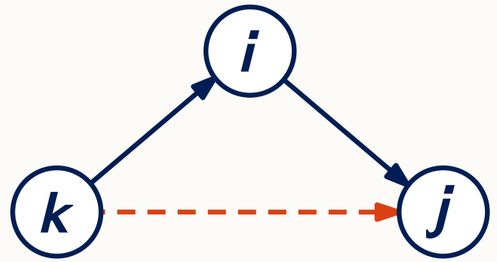

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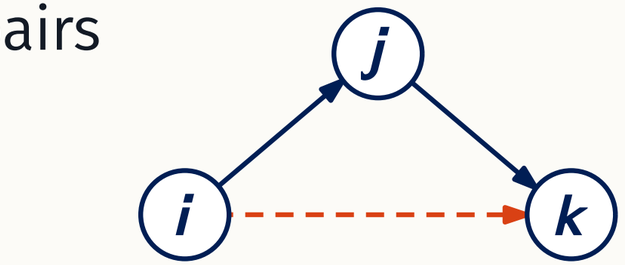

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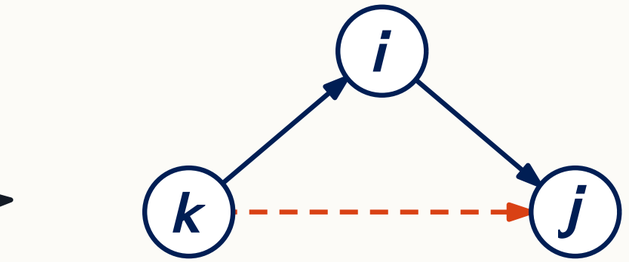

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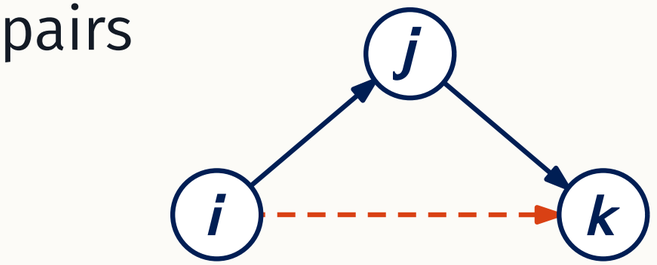

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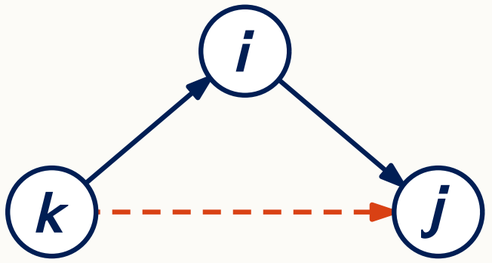

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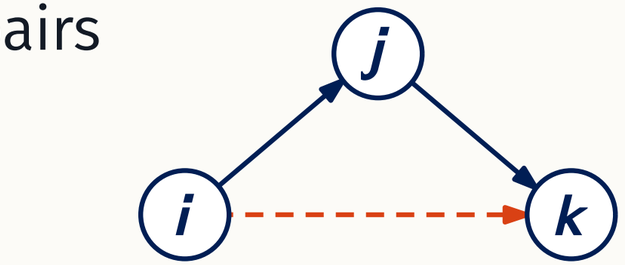

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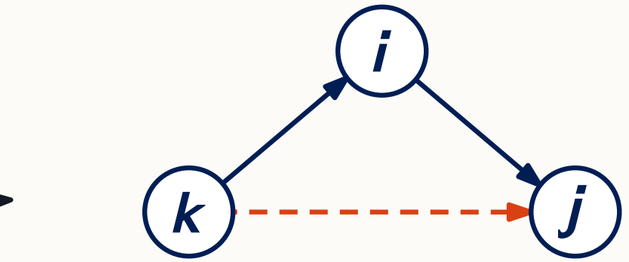

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Correctness follows from order.